The Beauty of Teaching Math

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& friends
Beauty in more ways than one

Beauty in

• Mathematics itself

• Ubiquity and criticality of mathematics

• Challenges in teaching and learning
  • How to engage students?
  • How to reduce attrition
  • How to deal with very diverse cultures and learning styles
What do I actually do in STEM teaching?

• Director Institute for Computational & Mathematical Engineering
  ICME runs 60 courses in computational mathematics and scientific computing on Stanford campus, serving over 3000 students

• Teach undergraduate and graduate courses in
  o Vector calculus
  o Linear algebra
  o Numerical PDEs
  o Energy systems

• Former middle school/high school teacher and tutor

• Active mentor of women and URM in STEM

• Active in outreach and mathematics evangelism
“When it comes down to it, it’s really all linear algebra”

Gene Golub

• Strong relevance across disciplines (cool factor)
• Applications used daily (search engines, for example)
• Lends itself both to verbal as well as visual learners
\[x + y = 2\]
\[y - z = 0\]
\[x + y + z = 3\]

what are \(x, y,\) and \(z\)?
It’s all about relations

- Predicting stock prices
- Computing heat flow in a theater
- Suggesting movies in Netflix
- Googling
- Optimizing design of an aircraft wing
- Positioning of goods in grocery store
- Analyzing Bergen’s chances for a sunny tomorrow
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1x + 1y + 0z &= 2 \\
0x + 1y + (-1)z &= 0 \\
1x + 1y + 1z &= 3
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1 & 1 & 0 \\
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1 & 1 & 1 \\
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MATRIX
Stanford

California

beating
Beating Eggs
Search engine term-document matrices allow teacher to throw some big (and exciting) numbers around.
Matrices to graphs – for visual learners
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Diagram:

- Node 1 is connected to Node 2.
- Node 3 is connected to Node 2.
- Node 4 is connected to Node 1 and Node 3.
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Ubiquity & Criticality

Exciting time for STEM/Comp Math
Pillar, elixir and glue
What motivates students to study math?

- Opens many doors / keeps doors open
- Versatility and agility
- Recognition that mathematics is foundational
- Desire to work in interdisciplinary teams
- Desire to help solve critical challenges
- Developing critical analysis skills
- Challenging oneself
- Money (particularly related to Data Science)

And of course students study math because it is required in engineering/sciences
Challenges

Strong diversity in classroom
Underrepresentation women and URM
Stress in students
Attrition
How to deal with strong diversity?

• Active vs passive learners
• Verbal vs visual learners
• Sequential vs global thinkers

• Different cultures and response to instructor
• Different levels of stress

• Different levels of preparation

Particularly critical in first quarter courses
What I try/do in the classroom re diversity

Active vs passive learners
- Interactive classroom style,
- with moments of individual introspection

Verbal vs visual learners
- Explain concepts in more than one way in class
- Provide additional and different materials than book

Helpful:
- Tutorials; office hours; online modules; discussion platforms
What I try/do in the classroom re diversity

Sequential vs global thinkers
• Present both top-down and bottom-up approaches
• Supply relevance, but do not rely on application knowledge in problem sets

Different cultures and response to instructor / stress
• Expectation survey at start of class
• Teaching assistants assigned to own group of students
• Midterm check-in with each student
• “WD40” for rusty students: bootcamps
What’s going on with women in math?

Currently in computational sciences (US):

- Undergraduates 20-25%
- MS/PhD 10-20%
- Faculty 2-12%
- SIAM membership 12%

Numbers have gone done since 80s (!)

Reported causes:

- stress
- family/work balance
- unconscious biasing
- subcritical mass
Stress in students is rampant

Particularly strong in first quarter of new degree
  • Adjusting to new learning and testing culture
  • Adjusting to being “average”

Poor preparation in primary and secondary education
  • Teaching to the test
  • Regurgitation, lack of creative thinking
  • Lack of grit

Increasing sense of entitlement
What causes students to drop out of math?

- Belief that they are just not any good
- Belief that they just cannot be any good, ever
- Fragile confidence – fear of failure

- Classroom atmosphere of judgment, not trust
- Lost love of learning – teaching to the test

- Little sense of usefulness, ubiquity
- Little sense of smashing careers in math fields
“I’m just not any good at mathematics”

“Innate ability” often stated early on & reinforced repeatedly

There is no math gene

*Just because some people can do it with little or no training,*

*it does not mean that others can not do it with training*

Stereotype threat

“Math is more challenging for women”

No evidence

Particularly harmful in US (low % women in comp math)

Imposter syndrome

**Growth mindset vs fixed mindset**
imposter syndrome
I am not as capable as people think I am

and they will find out sooner or later
It is more luck than talent that got me where I am
Stanford Survey

Conducted via Facebook friends and emailing lists

220 responses in 24 hours (80 male, 140 female)

Over 90% of answers from engineering/science fields
This applies to me never, rarely, sometimes, often or always:

2. I’m afraid of disappointing my advisor(s)

3. I often succeed on a task, even if I’m afraid I will not do well before I undertake it

4. I think it was luck more than talent that got me into grad school

5. I am afraid that my advisor/peers will find out that I am not as capable as they think I am

6. I give the impression that I am more capable than I really am

7. When I get complimented on a job well done, I often feel the person giving the compliment is merely trying to be nice

8. I am afraid that others will discover my lack of ability and/or skills

9. I compare my abilities to those around me and think they are more able than I am

10. If you answered often/always to any:
    - Do these factors affect your performance at school?
    - What, if anything, can your instructor(s)/advisor(s) do to help you deal better?
I’m afraid to be found out

I think that ....

often/always

43% male, 62% female

never/rarely

30% male, 15% female
I’m afraid to disappoint

I think that ..........

Often/always

40% male, 60% female

never/rarely

18% male, 6% female
Others are more capable

I think that ........

often/always
50% male, 71% female

never/rarely
22% male, 6% female
More by luck than talent

I think that ..... 

often/always
30% male, 36% female

never/rarely
42% male, 34% female
If you have such feelings, is performance affected?

Male
- 52% yes, negatively ("scared", "avoidance behavior")
- 27% yes, positively ("work harder")
- 21% no

Female
- 87% yes, negatively ("scared", "avoidance behavior", "exhaustion", "negative impacts on personal life")
- 7% yes, positively ("work harder")
- 7% no
Can anything be done?

Male

45% advisor/mentor/instructor can help
5% nothing can be done by anyone
50% I need to do this myself

Female

76% advisor/mentor/instructor can help
2% nothing can be done by anyone
11% I don’t know
11% I need to do this myself
What can be done?

Frequent suggestions for advisors

Male
• Give honest and regular feedback
• Give students a sense of importance of their work
• Be more involved

Female
• Set students up for (small) confidence building successes
• Be open about stress, I.S., own failures
• Give regular encouragement and positive reinforcement
Some concluding remarks
Teaching a second rank activity?

Teaching generally undervalued

• Not a significant part of tenure or promotions
• Few resources available for (re)design course material
• Best teaching practices infrequently shared
• Career instructors do not have a clear promotion path
New teaching approaches/formats

Flipped classroom – mixed responses
MOOCs – impact on campus students

Create more flexible, smaller teaching units
(1-unit classes and short courses, often online)
• Improves efficiency and reduces overlaps
• Allows for professional education, life-long learners
• Provides low-risk teaching opportunities for graduate students
Mindset

• Support the growth mindset – for students and yourself
• Trust, don’t judge – allow for (frequent) failure
• Reward progress – what matters is final mastery
**mAth**

**Anxiety**
- Detect symptoms of fixed mindset
- Bring students in at right level
- Set high standards, but show students how to reach them

**Agility**
- Deliver strong foundations – deep understanding
- Focus on love of learning
Symptoms of (partially) fixed mindset

• “This one low score shows I just cannot do it”
  Midquarter crisis, attrition

• “My low score is your fault”
  Strong resistance to admitting lack of understanding

• “There’s no point in studying”
  Need for study would show lack of innate ability
  Also fear of failure after studying

• “I’m good in calculus, just not in algebra”
  Confidence in one ability, not in another
**Tenacity**

- Emphasize that growth does not happen without effort
- Force/motivate students to work each day – bootcamp
- Encourage group work and interaction, cohort formation
Honest & constructive feedback

• Make students comfortable with mistakes
• Evaluate fairly - A’s for effort do not help in long run

High standards

• Set high standards – no need to cuddle students
• Show students how to reach standards and give them the support they need
Teach students to
• love challenges
• be intrigued by mistakes
• enjoy effort
• keep on learning

Beautifully argued in Mindset by Carol Dweck